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STABILITY AND NONLINEAR WAVY REGIMES IN DOWNWARD FILM FLOWS ON A CORRUGATED SURFACE

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The linear and nonlinear stability of downward viscous film flows on a corrugated surface to free-surface perturbations is analyzed theoretically. The study is performed with the use of an integral approach in ranges of parameters where the calculated results and the corresponding solutions of Navier–Stokes equations (downward wavy flow on a smooth wall and waveless flow along a corrugated surface) are in good agreement. It is demonstrated that, for moderate Reynolds numbers, there is a range of corrugation parameters (amplitude and period) where all linear perturbations of the free surface decay. For high Reynolds numbers, the waveless downward flow is unstable. Various nonlinear wavy regimes induced by varying the corrugation amplitude are determined.

Key words: *viscous film flow, corrugated surfaces, stability, nonlinear wavy regimes.*

1. INTRODUCTION AND FORMULATION OF THE PROBLEM

Theoretical investigations of film flows were started in [1], where an exact solution of Navier–Stokes equations was obtained for a free viscous thin-film flow down a smooth vertical wall:

$$U_0(y) = \frac{3\nu\,\mathrm{Re}}{H_0}\Big(\frac{y}{H_0} - \frac{y^2}{2H_0^2}\Big), \qquad H_0 = \Big(\frac{3\nu^2\,\mathrm{Re}}{g}\Big)^{1/3}.$$

Here $U_0(y)$ is the velocity profile in the film in the direction of the gravity force g, ν is the kinematic viscosity, and H_0 is the thickness of the liquid layer for a given mass-flux density $q_0 = \nu\,\mathrm{Re}$ (Re is the Reynolds number).

Subsequent theoretical and experimental studies showed that the Nusselt solution is almost never encountered in practice: normally, there are waves on the film surface. A great number of studies involved linear and nonlinear analyses of wave formation in a downward film flow on a smooth surface [2–7]. The problem of nonlinear waves on a film falling down a smooth plate has much in common with the problem of a steady viscous flow over a corrugated surface [8–15]. In both cases, the equations are essentially nonlinear, the shape of the free surface is unknown beforehand, surface-tension forces play an important role, and there is a spatial period involved in the problem. In spite of numerous applications of this problem to distillation processes and advanced heat exchangers [16], there are few experimental [8, 17] and theoretical [9–15] studies of the film flow on a corrugated surface. For instance, the film flow down a sine-shaped surface with a small corrugation amplitude, as compared with the Nusselt thickness of the film, was examined with the perturbation technique by Wang [9]. Kang and Chen [10] extended this approach to the case of a two-layer film flow along a corrugated surface with a small corrugation amplitude. Pozrikidis [14] considered a creep flow down a curved inclined surface, using the boundary-element method and disregarding inertial forces. The asymptotic approach was used by Shetty and Cerro [12] to examine a liquid flow down a corrugated surface with a corrugation amplitude much greater than the Nusselt film thickness H_0. Treating corrugation in the linear approximation, Bontozoglou and Papapolymerou [13] examined resonant effects in the range of finite Reynolds numbers. The numerical solution of Navier–Stokes equations allowed the present author [14] to study

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the film flow in the range of finite Reynolds numbers for corrugation amplitudes commensurable with the Nusselt thickness. In [15], the present author considered a film flow down complex-shaped three-dimensional surfaces with rough corrugation (with an amplitude much greater than the Nusselt thickness) and fine texture (with an amplitude commensurable with H_0). All theoretical works mentioned above dealt with a waveless film flow down a corrugated surface (in essence, an analog of the Nusselt solution for a film flow down a smooth wall). The present work is aimed at studying stability of a film flow down a corrugated surface to free-surface perturbations and calculating wavy flow regimes down such surfaces.

2. GOVERNING EQUATIONS

2.1. Waveless Solutions. The waveless film flow down a one-dimensional corrugated surface is described by a system of Navier–Stokes equations with appropriate boundary conditions, which was described in detail in [14, 15]. The solutions are presented in the form

$$u(x,\eta) = \frac{1}{2}U_1(x) + \sum_{m=2}^{M} U_m(x)T_{m-1}(\eta_1), \qquad \eta_1 = 2\eta - 1,$$
$$U_m(x) = U_m^0 + \sum_{\substack{n=-N/2+1 \\ n\neq 0}}^{N/2-1} U_m^n \exp\frac{2\pi i n x}{L}, \qquad (U_m^{-n})^* = U_m^n, \qquad m = 1,\dots,M, \tag{2.1}$$

where $u(x,\eta)$ is the velocity-vector component aligned with the gravity force, x is the coordinate along the gravity-force direction, $\eta = (y - f(x))/H(x)$, y is the coordinate in the direction perpendicular to the gravity force, L is the corrugation period, $f(x)$ is the corrugation shape function, $H(x) = h(x) - f(x)$ is the local film thickness, $h(x)$ is the shape function of the free surface, and $T_m(\eta_1)$ are the Chebyshev polynomials; the asterisk denotes complex conjugation.

The problem reduces to a system of nonlinear equations for determining the harmonics U_m^n and is solved numerically. The velocity field in the y direction, the pressure in the film, and the shape of the free surface are uniquely reconstructed from the harmonics U_m^n.

In examining the wave dynamics of the film flow down a smooth surface, an integral approach (set of Shkadov's equations [2]), which implies long-wave perturbations, is frequently used. This approach was extended in [14] to a film flow down a corrugated surface with a corrugation period much greater than the film thickness ($\varepsilon = H_0/L \ll 1$). The basic idea of the integral approach consist in using a self-similar profile of streamwise velocity

$$u(x,y) = \frac{3\nu\,\mathrm{Re}}{H(x)}\Big(\frac{y - f(x)}{H(x)} - \frac{(y - f(x))^2}{2H^2}\Big). \tag{2.2}$$

After integration across the film and with allowance for Eq. (2.2), we obtain the following system of equations for the dynamics of the film flow on a corrugated surface [14, 15]:

$$\frac{\partial q}{\partial t} + \frac{6}{5}\frac{\partial}{\partial x}\frac{q^2}{H} = \frac{3}{\varepsilon\,\mathrm{Re}}\Big(H - \frac{q}{H^2}\Big) + \varepsilon^2 \mathrm{We}\, H\Big(\frac{\partial^3 H}{\partial x^3} + \frac{1}{\varepsilon_1}\frac{d^3 f}{dx^3}\Big),$$
$$\frac{\partial H}{\partial t} + \frac{\partial q}{\partial x} = 0. \tag{2.3}$$

Here $q(x,t)$ is the instantaneous flow rate in the film normalized to q_0; the film thickness is normalized to H_0; the length scale in the x direction is the corrugation period; the time scale is H_0L/q_0; $\mathrm{We} = (3\mathrm{Fi})^{1/3}/\,\mathrm{Re}^{5/3}$ is the Weber number, $\mathrm{Fi} = (\sigma/\rho)^3/(g\nu^4)$ is the film number, ρ is the liquid density, σ is the surface tension, and $\varepsilon_1 = H_0/A$ (A is the corrugation amplitude). Note that the presence of the capillary term in Eq. (2.3) is attributed to high values of the film number Fi in the Weber criterion for most liquids being used.

The waveless flow down a corrugated surface is described by steady solutions of system (2.3): $H = H_b(x)$, $q = 1$. To find these solutions numerically, we used the Newton method and the Fourier expansion:

$$H(x) = \sum_{n=-N/2+1}^{N/2-1} H_n \exp(2\pi i n x), \qquad (H_{-n})^* = H_n.$$

2.2. Stability of Steady Solutions. Substituting $q = 1 + q'$ and $H = H_b(x) + H'(x,t)$ into system (2.3) and linearizing the resultant equations in the neighborhood of the original equations, we obtain a system of equations with periodically changing coefficients:

$$\frac{\partial q'}{\partial t} + \frac{12}{5}\frac{\partial}{\partial x}\frac{q'}{H_b} - \frac{6}{5}\frac{\partial}{\partial x}\frac{H'}{H_b^2} = \frac{3}{\varepsilon \operatorname{Re}}\Big(H' - \frac{q'}{H_b^2} + \frac{2H'}{H_b^3}\Big) + \varepsilon^2 \mathrm{We}\, H' \frac{d^3 h_0}{dx^3} + \varepsilon^2 \mathrm{We}\, H_b \frac{d^3 H'}{dx^3},$$
$$\frac{\partial H'}{\partial t} + \frac{\partial q'}{\partial x} = 0, \qquad h_0 = H_b(x) + \frac{1}{\varepsilon_1} f(x). \tag{2.4}$$

According to Floquet's theorem, the solutions of Eqs. (2.4) bounded in terms of the x coordinate are presented as

$$H' = \psi(x)\exp(-\gamma t + 2\pi i Q x) + \overline{\psi(x)}\exp(-\bar\gamma t - 2\pi i Q x),$$
$$q' = \varphi(x)\exp(-\gamma t + 2\pi i Q x) + \overline{\varphi(x)}\exp(-\bar\gamma t - 2\pi i Q x), \tag{2.5}$$
$$\psi(x) = \psi(x+1), \qquad \varphi(x) = \varphi(x+1).$$

The bar denotes complex conjugation; Q is a real parameter varying from zero to unity. Substituting Eqs. (2.5) into Eqs. (2.4), we obtain the following problem with eigenvalues:

$$2\pi i Q\varphi + \frac{d\varphi}{dx} = \gamma\psi,$$
$$\Big(C_0(x) + C_1(x)\frac{d}{dx} + C_2(x)\frac{d^2}{dx^2} + C_3(x)\frac{d^3}{dx^3}\Big)\psi + \Big(D_0(x) + D_1(x)\frac{d}{dx}\Big)\varphi = \gamma\varphi,$$
$$C_0(x) = G_3(x) + 2\pi i Q G_4(x) - (2\pi i Q)^3 G_5(x), \quad C_1(x) = G_4(x) - 3(2\pi i Q)^2 G_5(x),$$
$$C_2(x) = -3(2\pi i Q) G_5(x), \quad C_3(x) = -G_5(x), \quad D_0(x) = G_1(x) + (2\pi i Q) G_2(x), \tag{2.6}$$
$$D_1(x) = G_2(x), \quad G_1(x) = \frac{12}{5}\frac{d}{dx}\frac{1}{H_b} + \frac{3}{\varepsilon \operatorname{Re} H_b^2}, \quad G_2(x) = \frac{12}{5}\frac{1}{H_b}, \quad G_4(x) = -\frac{6}{5}\frac{1}{H_b^2},$$
$$G_3(x) = -\frac{6}{5}\frac{d}{dx}\frac{1}{H_b^2} - \frac{3}{\varepsilon \operatorname{Re}}\Big(1 + \frac{2}{H_b^3}\Big) - \varepsilon^2 \mathrm{We}\frac{d^3 h_0}{dx^3}, \qquad G_5(x) = \varepsilon^2 \mathrm{We}\, H_b.$$

Here the coefficients and eigenfunctions are periodic over the x coordinate:

$$\varphi(x) = \sum_{n=-N/2+1}^{N/2-1} \varphi_n \exp(2\pi i n x), \qquad \psi(x) = \sum_{n=-N/2+1}^{N/2-1} \psi_n \exp(2\pi i n x),$$
$$C(x) = \sum_{n=-N/2+1}^{N/2-1} C_n \exp(2\pi i n x). \tag{2.7}$$

The number of harmonics N in Eqs. (2.7) corresponds to the number of harmonics in the expansion of the basic solution. Substituting Eqs. (2.7) into Eqs. (2.6), we obtain a problem of determining the eigenvalues for a complex matrix of the general form [of size $2(N-1) \times 2(N-1)$], which is solved numerically. To study stability of the solution $H_b(x)$, we have to analyze $2(N-1)$ eigenvalues for each value of the parameter Q in the range $[0,1]$. The solution is stable if the real parts of all $2(N-1)$ eigenvalues are greater than zero or equal to zero for all $Q \in [0,1]$. In this case, all perturbations of the free surface decay (or do not grow) with time.

Perturbations (2.5) with the parameter $Q = 0$ are worth mentioning. Such perturbations have the same period as the original solution. Instability to this class of perturbations means that the waveless regime of the flow down a corrugated surface is impossible. Regimes unstable to perturbations with finite values of Q can be observed in some regions until unstable perturbations are developed.

For a zero value of the corrugation amplitude ($A = 0$), problem (2.6) reduces to the known problem of stability of the Nusselt solution for a film flow down a smooth wall. In this case, the parameter L has no physical meaning (only a scale), and all perturbations with the wave length L/Q greater than a certain value (λ_*) increase with time:

$$\frac{L}{Q} > \lambda_* = 2\pi H_0 \sqrt{\frac{\mathrm{We}}{3}} = 2\pi \Big(\frac{3\nu^2}{g}\Big)^{1/3} \Big(\frac{\mathrm{Fi}}{9}\Big)^{1/6} \frac{1}{\sqrt{\mathrm{Re}}}, \qquad Q \in [0, 1]. \tag{2.8}$$

In the present work, the calculations were performed for two liquids: nitrogen at the saturation line [$\mathrm{Fi} = 1.231 \cdot 10^{11}$, $(3\nu^2/g)^{1/3} = 0.0216$ m, and $\lambda_* = 6.65$, 2.71, and 2.35 mm for $\mathrm{Re} = 1$, 6, and 8, respectively] and water–glycerin mixture [3] [$\mathrm{Fi} = 7.249 \cdot 10^6$, $(3\nu^2/g)^{1/3} = 0.251$ mm, and $\lambda_* = 15.225$, 8.790, and 3.400 mm for $\mathrm{Re} = 1$, 3, and 20, respectively].

2.3. Wavy Regimes of the Film Flow Down a Corrugated Surface. The form of perturbations in (2.5) implies that there are two different periods in the x direction in the general case. The first period corresponds to the corrugation period of the wall L, and the second period is L/Q. For low values of Q, we have "long-modulated" perturbations. Perturbations with $\mathrm{Real}\,\gamma = 0$ are neutral. At these values of parameters, the wavy regime branches off from the original solution. The new solution is presented as a double Fourier series

$$H(\xi, x) = \sum_{n=-N/2+1}^{N/2-1} \sum_{m=-M/2+1}^{M/2-1} H_{nm} \exp(2\pi i n \xi) \exp(2\pi i m x),$$

$$q(\xi, x) = \sum_{n=-N/2+1}^{N/2-1} \sum_{m=-M/2+1}^{M/2-1} q_{nm} \exp(2\pi i n \xi) \exp(2\pi i m x), \tag{2.9}$$

$$(H_{-n,-m})^* = H_{nm}, \qquad (q_{-n,-m})^* = q_{nm}, \qquad \xi = x - ct.$$

The phase velocity c here is the eigenvalue of the nonlinear problem. The branching wavy regimes are described by the following equations:

$$-Qc \frac{\partial q}{\partial \xi} + \frac{6}{5} \frac{\partial}{\partial x} \frac{q^2}{H} + \frac{6}{5} Q \frac{\partial}{\partial \xi} \frac{q^2}{H} = \frac{3}{\varepsilon \,\mathrm{Re}} \Big(H - \frac{q}{H^2}\Big) + \varepsilon^2 \mathrm{We}\, H \Big(\frac{\partial^3 H}{\partial x^3} + 3Q \frac{\partial^3 H}{\partial x^2\, \partial \xi}$$

$$+ 3Q^2 \frac{\partial^3 H}{\partial x\, \partial \xi^2} + Q^3 \frac{\partial^3 H}{\partial \xi^3} + \frac{1}{\varepsilon_1} \frac{d^3 f}{dx^3}\Big), \tag{2.10}$$

$$-Qc \frac{\partial H}{\partial \xi} + \frac{\partial q}{\partial x} + Q \frac{\partial q}{\partial \xi} = 0.$$

The phase of one harmonic in Eqs. (2.9) can be assumed to be known (e.g., $H_{10} = 0$). As Eqs. (2.10) are symmetric with respect to the transformation $\xi \to \xi + \mathrm{const}$, the origin may be chosen arbitrarily. This circumstance allows us to determined the unknown phase velocity c in addition to other harmonics. The second equation in (2.10) and the normalization condition yield

$$q_{nm} = \begin{cases} nQcH_{nm}/(nQ + m), & n \neq 0, \\ 0, & n = 0,\ m \neq 0, \\ 1, & n = 0,\ m = 0. \end{cases}$$

Thus, the problem reduces to finding the unknown harmonics H_{nm} and phase velocity c and is solved numerically by the Newton method. Note that biperiodic wavy solutions for a flow down a smooth wall were obtained in [6].

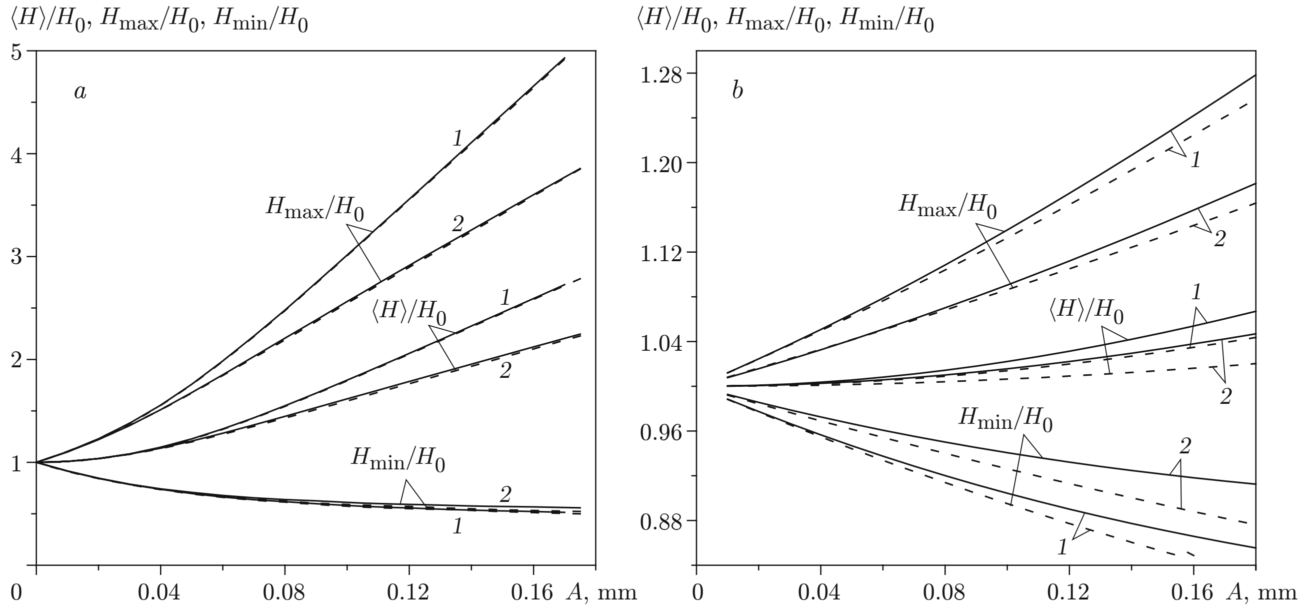

Fig. 1. Liquid nitrogen (a) and water–glycerin (b) film flow down a corrugated surface with a corrugation period $L = 1.57$ mm for $\mathrm{Re} = 5$ (1) and 20 (2): the solid and dashed curves refer to the calculations by the Navier–Stokes equations and by the integral model, respectively.

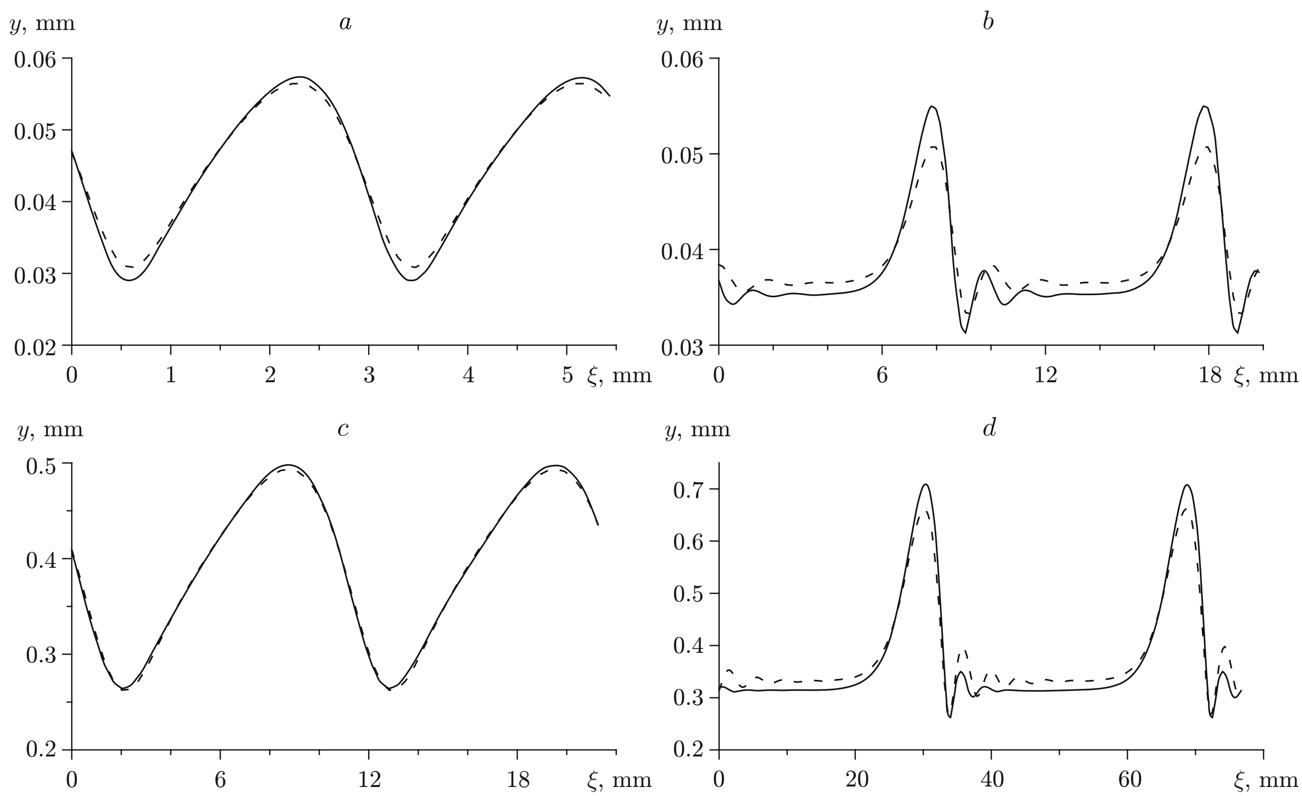

Fig. 2. Liquid nitrogen (a and b) and water–glycerin (c and d) wavy film flow down a smooth surface: (a) $\mathrm{Re} = 10$ and $\lambda = 2.85$ mm; (b) $\mathrm{Re} = 6$ and $\lambda = 10$ mm; (c) $\mathrm{Re} = 4.2$ and $\lambda = 10.8$ mm; (d) $\mathrm{Re} = 4.2$ and $\lambda = 38.5$ mm; the solid and dashed curves refer to the calculations by the Navier–Stokes equations and by the integral model, respectively.

3. CALCULATION RESULTS

3.1. Waveless Solutions for a Film Flow Down a Corrugated Surface. Comparison of the Integral Approach with the Solutions of the Navier–Stokes Equations. The results calculated by the Navier–Stokes equations and by the integral model for two liquids are plotted in Fig. 1. For a fixed corrugation period, we calculated the velocity fields (2.1) and the shape of the free surface $h(x)$ for different values of the corrugation amplitude. The corrugation shape function in all calculations was defined as $f(x) = 0.5(1 - \cos(2\pi x))$. Note that the corrugation parameters considered here are close to the corresponding characteristics of the fine texture for elements of the commercial setup Sulzer 500Y [16]. The basic characteristics of the free surface (the mean film thickness $\langle H \rangle$ and the maximum and minimum local film thicknesses $H_{\max}$ and $H_{\min}$) were calculated for a low-viscosity liquid (nitrogen at the saturation line at atmospheric pressure) and for a water–glycerin solution [3] for $\mathrm{Re} = 5$ and 20. The solutions of the Navier–Stokes equations for steady traveling waves and the corresponding solutions of the integral model (2.3) are plotted in Fig. 2 for the film flow down a smooth surface ($A = 0$). The shape of the free surface was calculated for two liquids and for two types of waves [3, 5].

A comparison of the solutions of the Navier–Stokes equations and the corresponding solutions (2.3) for two limiting cases allow us to conclude that the integral approach is suitable for analyzing wave formation in a flow down a corrugated surface.

3.2. Linear Stability of the Film Flow Down a Corrugated Surface to Perturbations of the Free Surface. The results on linear stability are plotted in Figs. 3–6. For comparatively moderate Reynolds numbers, there exist corrugation parameters that ensure a stable film flow to arbitrary pertubations of the free surface (the values of $Q \in [0, 1]$ in (2.5) were varied). For instance, the range of these parameters for a liquid nitrogen film flow in Fig. 3a for $\mathrm{Re} = 1$ is located between curves 1 and $1'$. Similar ranges for the water–glycerin film flow are shown in Fig. 3b. Note that there are unstable perturbations of the free surface for all Reynolds numbers in the flow down a smooth wall.

In a certain sense, wall corrugation exerts a stabilizing effect on evolution of perturbations on the free surface. Figure 4 shows the curves where the flow loses its stability to free-surface perturbations with a period equal to the corrugation period [$Q = 0$ in (2.5)]. On these curves, the real part of one pair of complex-conjugate eigenvalues vanishes. Note that waveless solutions, which are unstable to perturbations with $Q = 0$, cannot be observed in experiments. The corrugation parameters for such solutions in the case of a liquid nitrogen film flow in Fig. 4a are located above curves 1–5. The corrugation period here is normalized to the wave length of the neutral perturbation (2.8) in the film flow down a smooth wall. Similar results for a water–glycerin film flow are plotted in Fig. 4b. Figure 5 shows the ranges of corrugation parameters where the corresponding waveless solutions are stable to perturbations with a period equal to the corrugation period (below curve 1) and the range where the waveless solutions are stable to arbitrary perturbations of the free surface (region bounded by curves 2 and $2'$). The calculation results plotted in Fig. 5 refer to the liquid nitrogen film flow with $\mathrm{Re} = 5$. In regions between curves 1 and 2 and below curve $2'$, unstable perturbations are those with finite values of the parameter Q in (2.5). Figure 6 shows such values of Q (in the regions located below the corresponding curves) for three values of the corrugation amplitude.

The analysis performed allows us to draw the following conclusions. There exist corrugation parameters at which the waveless solution (analog of the Nusselt solution for a film flow down a smooth wall) may be unstable to perturbations with the same period and is not observed in any region of the flow) (in contrast to the Nusselt solution observed in the initial region of the flow). At the same time, there exist corrugation parameters at which the waveless solution is stable to arbitrary perturbations of the free surface for moderate Reynolds numbers (in this case, corrugation of the wall exerts a stabilizing effect). For other values of the corrugation parameters and Reynolds numbers, the waveless solution is unstable to perturbations with finite values of Q and may be observed only in the initial region of the flow (similar to the Nusselt solution in the case of a film flow down a smooth wall).

3.3. Calculation of Wavy Regimes of the Film Flow Down a Corrugated Surface. To describe nonlinear wavy regimes of the film flow down a smooth wall within the framework of Eqs. (2.3), it suffices to use only one external parameter $\mathrm{Re}/\mathrm{Fi}^{1/11}$. For each value of this parameter, there are many different one-parameter families of steady traveling solutions [5]. The internal parameter for each family in the wave length λ (or the wave number $\alpha = 2\pi/\lambda$). Each family of solutions exists in a certain range of wave lengths (normally, $0 < \alpha < \alpha_*$) and is

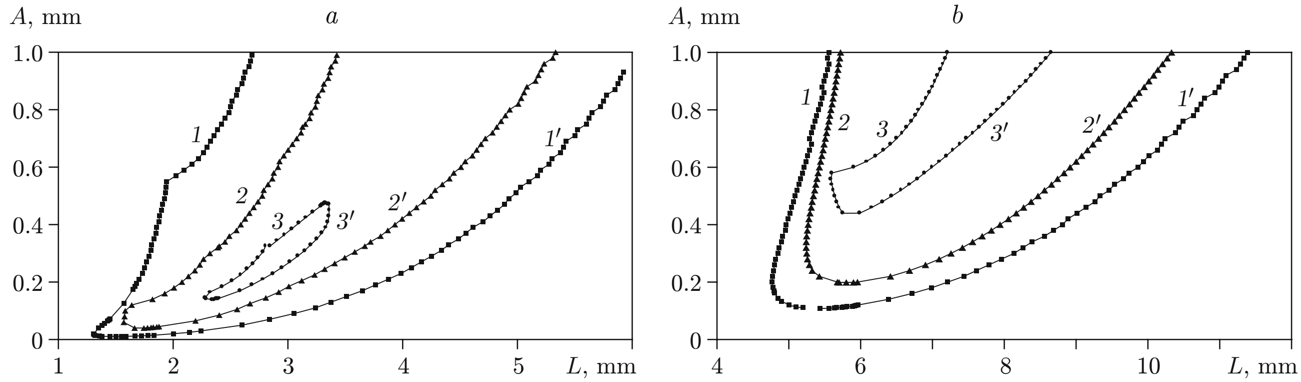

Fig. 3. Corrugation parameters that ensure a stable film flow down a corrugated wall with respect to all linear perturbations of the free surface (regions between the corresponding curves): (a) liquid nitrogen for Re = 1 (1 and 1′), 5 (2 and 2′), and 8 (3 and 3′); (b) water–glycerin solution for Re = 0.5 (1 and 1′), 1 (2 and 2′), and 3 (3 and 3′).

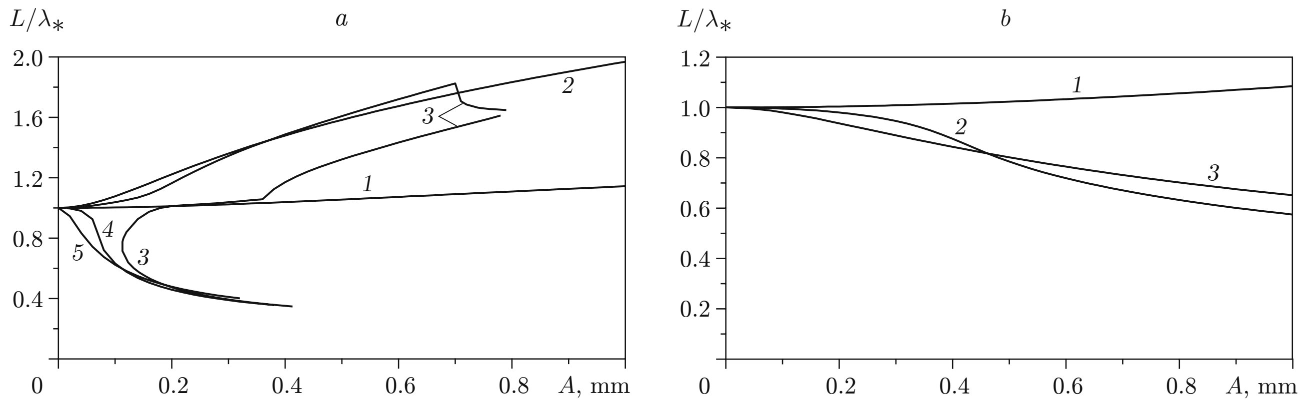

Fig. 4. Corrugation parameters that ensure a stable film flow down a corrugated wall with respect to perturbations with a period equal to the corrugation period (regions below and between the corresponding curves): (a) liquid nitrogen for Re = 1 and $\lambda_* = 6.65$ mm (1), Re = 6 and $\lambda_* = 2.71$ mm (2), Re = 8 and $\lambda_* = 2.35$ mm (3), Re = 10 and $\lambda_* = 2.1$ mm (4), and Re = 20 and $\lambda_* = 1.49$ mm (5); (b) water–glycerin solution for Re = 3 and $\lambda_* = 8.79$ mm (1), Re = 5 and $\lambda_* = 6.81$ mm (2), and Re = 20 and $\lambda_* = 3.4$ mm (3).

combined with other families in a complicated manner. The condition of stability to perturbations with the same period significantly decreases the number of possible families of solutions, because most of them to not contain regimes stable to this class of perturbations. Two families of solutions are specific in terms of stability. The first family (see Fig. 2a and c) branches off from the trivial solution $H(x) = 1$. The second family of solutions branches off from the first one (see Fig. 2b and d).

An analysis of wavy regimes of the film flow down a corrugated surface reveals new external parameters: corrugation period L and amplitude A. The period L/Q in (2.9) is an internal parameter, similar to the wave length λ in the case of the wavy film flow down a smooth wall. Thus, with allowance for the number of external parameters, the case considered is substantially more complicated (e.g., the wavy flow down a smooth wall with all possible solutions is only a simple limiting case of this problem). The calculated results, nevertheless, give a "simpler" wave pattern for a film flow down a corrugated surface. In the present work, we consider only the wavy flow of a liquid nitrogen film. No different families of wavy solutions were found for high corrugation amplitudes. The calculations were started from low corrugation amplitudes [the initial approximation for solving Eq. (2.10) was the corresponding wavy solutions for a film flow down a smooth wall]. Figure 7 shows the typical instantaneous profiles of the corrugated free surface with different corrugation amplitudes (note that the corrugation period in the x direction is equal to Q in the chosen coordinates). The calculations started from solutions for "long" waves of the

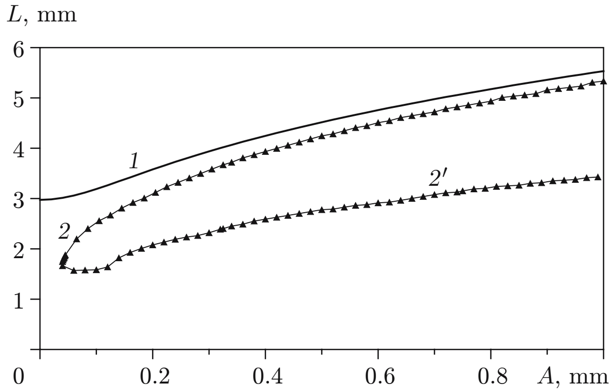

Fig. 5

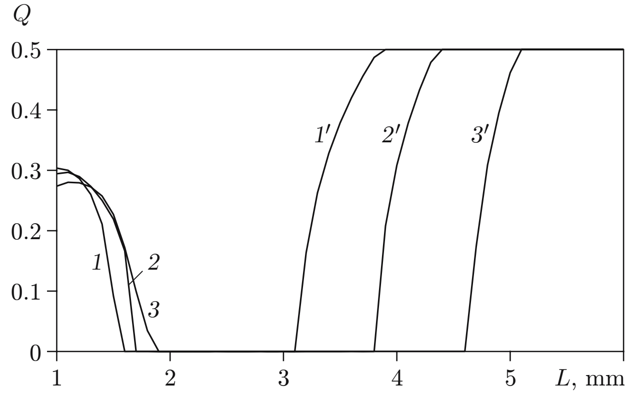

Fig. 6

Fig. 5. Corrugation parameters that ensure a stable liquid nitrogen film flow with Re = 5 down a corrugated surface with respect to perturbations with a period equal to the corrugation period (region below curve 1) and to all linear perturbations of the free surface (region between curves 2 and 2′).

Fig. 6. Liquid nitrogen film flow down a corrugated wall for Re = 1 and $A = 0.1$ (1 and 1′), 0.2 (2 and 2′), and 0.4 mm (3 and 3′); the regions below the curves are regions of unstable perturbations.

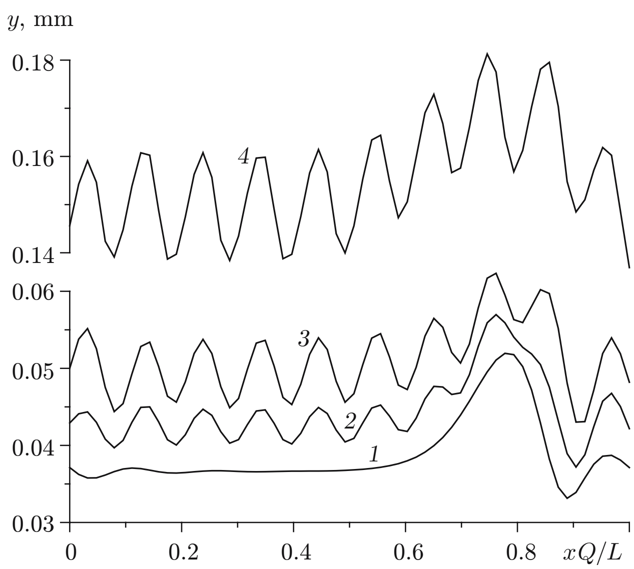

Fig. 7. Wavy flow of the liquid nitrogen film down a corrugated wall for Re = 6, $L = 1.2$ mm, $Q = 0.1035$, and $A = 0$ (1), 0.1 (2), 0.2 (3), and 0.135 mm (4).

second family. Similar calculations were performed for different values of Re, L, and Q and started from different wavy solutions for low corrugation amplitudes (including the solutions for "short" waves of the first family). In some cases, solutions for high corrugation amplitudes ($A \approx 0.2$ mm) could be obtained; in other cases, the solution degenerated into a waveless solution; and sometimes merging with other wavy solutions was observed (e.g., those started from low corrugation amplitudes from solutions for waves from other families). Other parameters, apart from the corrugation amplitude, were also varied. In this case, merging of different solutions and degeneration into waveless solutions could also occur. Thus, one set of parameters (Re, A, L, and Q) could be obtained by different methods, avoiding bifurcation points. No different families of solutions were observed for high corrugation amplitudes. In this sense, the wave pattern becomes simpler. The wavy solutions for different values of Q ("long" and "short" waves) are plotted in Figs. 8 and 9 for fixed values of the corrugation amplitude, corrugation period, and Reynolds number.

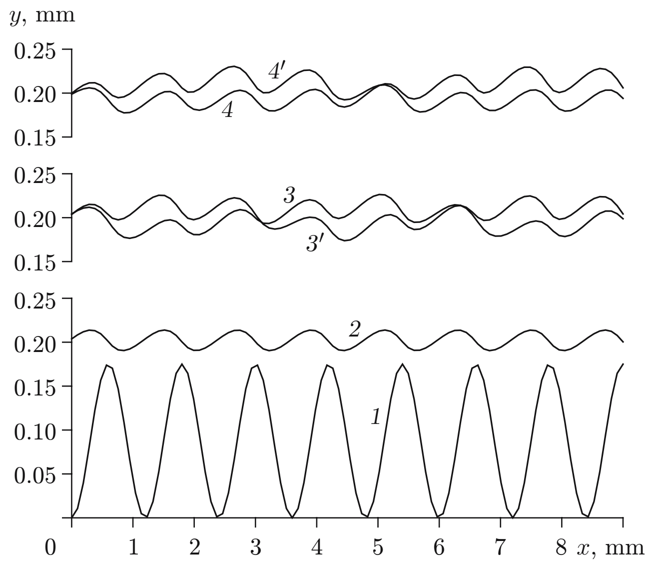

Fig. 8

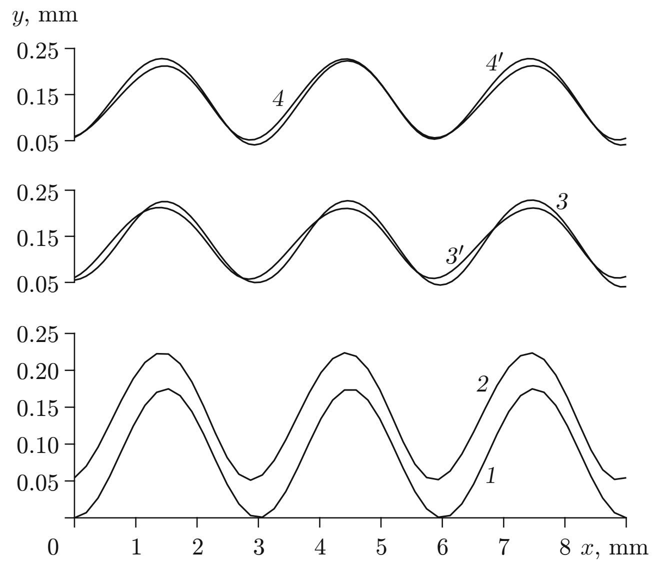

Fig. 9

Fig. 8. Wavy flow of a liquid nitrogen film down a corrugated wall for $\mathrm{Re} = 10$, $A = 0.175$ mm, and $L = 1.2$ mm: profile of the corrugated wall (1), waveless solution (2), "long" waves with $Q = 0.12$ for $t = 0$ (3) and $t = T/2$ (3′), and "short" waves with $Q = 0$ for $t = 0$ (4) and $t = T/2$ (4′).

Fig. 9. Wavy flow of a liquid nitrogen film down a corrugated wall for $\mathrm{Re} = 10$, $A = 0.175$ mm, and $L = 3$ mm: profile of the corrugated wall (1), waveless solution (2), "short" waves with $Q = 1$ for $t = 0$ (3) and $t = T/2$ (3′), and "long" waves with $Q = 0.5$ for $t = 0$ (4) and $t = T/2$ (4′).

The analysis performed allows us to conclude that wall corrugation exerts a significant effect on the global pattern of nonlinear waves on the free surface. For high corrugation amplitudes, nonuniqueness of the families of wavy solutions disappears.

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